Int J Theor Phys (2009) 48: 2256–2261
DOI 10.1007/s10773-009-0013-2

Neutrino Oscillation in the Space-Time with a Global Monopole

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Received: 10 February 2009 / Accepted: 6 April 2009 / Published online: 11 April 2009

Abstract The mass neutrino interference phase in a global monopole space time along the null trajectory and the geodesic is studied, and we find that the conserved energy changes a factor when a particle travels along the geodesic, if compared with the energy in the space time without the global monopole. The oscillation phase is increased by a factor due to the correction of the global monopole, comparing with the case in Schwarzschild space time. We obtain that the type-I phase along both the null and geodesic has a difference of a factor of $1 - 8\pi\eta^2$, and that the phase along the geodesic is the double of that along the null.

Keywords Gravitational field · General relativity · Neutrino interference phase · Global monopole space-time · Geodesic line

1 Introduction

Since the confirmation of mass neutrino by Super-Kamiokande atmospheric neutrino experiment [1], the consideration of the mass neutrino oscillations [2, 3] has been a hot topic. In the theoretical study, the neutrino oscillations in the flat space time were extended to the case in a gravitational field space-time [4]. Cardall and Fuller have developed a simple formalism for treating neutrino oscillations in a curved space-time [5]. In 2001, Zhang and Beesham also studied the propagation of mass neutrinos in Schwarzschild space-time with a detailed scheme, and they calculated the interference phase along the geodesic line, which will produce a factor 2 when compared to the value along the null line [6]. In 2003, they also analyzed in detail the mass neutrino phase along the geodesic line and the null in curved and flat space-time, separately [7]. This issue of the factor 2 can be solved if the two neutrino arrival time difference is taken into account [6–8]. As an extended study of mass neutrino, the gravitational effect on the neutrino oscillation has attracted much attention recently [9–12]. Moreover, the rotational inertial effect on mass neutrino oscillation is also

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called attention [13]. Furthermore, many alternative mechanisms have been proposed to account for the gravitational effect on the neutrino oscillation [14–16]. As a further theoretical exploration, neutrino oscillation in space-time with both curvature and torsion [17–19] has been proposed.

In 2008, the researches about the neutrino oscillation have new progress. Cuesta and Lambiase study the neutrino mass spectrum [20]. They introduce a model-independent novel nonpareil method to overcome difficulties of the issue of determining the ν masses from solar or atmosphere ν experiments concerning the ability of ν detectors to be sensitive to the species mass-square difference instead of so doing to the electron neutrino ν mass itself. Akhmedov, Maltoni and Smirnov present the neutrino oscillograms for different oscillation channels and discuss the effects of non-vanishing 1-2 mixing and CP-violation [21].

In this paper, we extend the work in Schwarzschild space-time to the space-time with a global monopole and study its effect on the interference phase, where the space-time has the topological defect. The global monopole has an effect on the particle orbits and Hawking radiation [22, 23]. Because of the existence of the scale of a gauge-symmetry breaking η, boundedness and stability threshold for circle orbits scales up by $(1-8\pi\eta^2)^{-1}$, perihelion shift and light bending by $(1-8\pi\eta^2)^{-3/2}$, while the Hawking temperature scales down by $(1-8\pi\eta^2)^2$ comparing with the Schwarzschild case [22]. In addition, the energy of a particle traveling along the geodesic is also conserved in this case. The conserved energy, however, will be changed by a factor, $1-8\pi\eta^2$, comparing with the cases in the space-time without the global monopole. It leads to a change in the interference phase of the mass neutrino. We calculate the phase along the null trajectory and along the radical geodesic. The phase changes the same factor along the geodesic as it along the null trajectory. So, the geodesic phase keeps double of the result along the null, which is in agreement with the cases in the space time without the global monopole.

The paper is organized as the follows. In Sect. 2, we introduce the line element of the space-time with a global monopole and obtain the tetrad. In Sects. 3 and 4, we calculate the neutrino phase along the null and geodesic, respectively. At last, the conclusion and discussion are given. Throughout the paper, the units $G=c=\hbar=k_B=1$ are used.

2 Space-Time with a Global Monopole

The line element of black holes with a global monopole is described by [22]

$$ds^2=\left(1-8\pi\eta^2-\frac{2M}{r}\right)dt^2-\left(1-8\pi\eta^2-\frac{2M}{r}\right)^{-1}dr^2-r^2(d\theta^2+\sin^2 d\varphi^2) \quad (1)$$

where η is the scale of a gauge-symmetry breaking. The event horizon is given by $2M(1-8\pi\eta^2)^{-1}$. There are some effects of the global monopole charge on the particle orbits and Hawking radiation. It turns out that the existence, boundedness and stability threshold for circle orbits scales up by $(1-8\pi\eta^2)^{-1}$, perihelion shift and light bending by $(1-8\pi\eta^2)^{-3/2}$, while the Hawking temperature scales down by $(1-8\pi\eta^2)^2$ when compared to the Schwarzschild case [22].

Along the geodesic, we can define a constant

$$E\equiv g_{ab}\left(\frac{\partial}{\partial t}\right)^a\left(\frac{\partial}{\partial\tau}\right)^b=g_{00}\frac{dt}{d\tau}=\left(1-8\pi\eta^2-\frac{2M}{r}\right)\frac{dt}{d\tau}, \quad (2)$$

where $(\frac{\partial}{\partial\tau})^b$ is the tangent vector of the geodesic [24]. If the mass of a particle is m, the 4-velocity and 4-momentum are $U^a = (\frac{\partial}{\partial\tau})^a$, $P^a = mU^a$, respectively. If p is a point on the geodesic and G is a static observer (its 4-velocity is Z^a) passing the point p, the E defined in (2) can be rewritten as

$$E = \xi_a U^a = \frac{1}{m}\xi_a P^a = \frac{(g_{00})^{1/2}}{m} Z_a P^a = \frac{(g_{00})^{1/2}}{m} E_{local}, \tag{3}$$

where $E_{local} = Z_a P^a$ is the energy of the local measurement by the observer; $\xi^a = (\partial/\partial t)^a$ is the static killing vector field. E is the energy of per unit rest mass measured by the observer at infinity. It is obvious that E is a constant along the geodesic, while E_{local} is not. So, the conserved energy of a particle with mass m traveling along the geodesic is

$$E_0 = m g_{00} \frac{dt}{d\tau}. \tag{4}$$

In the space time with a global monopole, $r \to \infty$, $g_{00} \to 1 - 8\pi\eta^2$, the conserved energy scales down by $a^2 = 1 - 8\pi\eta^2$ comparing with the Schwarzschild value. It is this difference that influences on the phase.

In the spherical, static and isotropic coordinate system $X^1 = \rho\sin\theta\cos\phi$, $X^2 = \rho\sin\theta\sin\phi$, $X^3 = \rho\cos\theta$, the tetrad of the space-time with a global monopole is given from [25, 26]

$$ds^2 = C(\rho)dt^2 - D(\rho)(d\rho^2 + \rho^2 d\Omega^2). \tag{5}$$

The tetrad is

$$\begin{pmatrix} \sqrt{C} & 0 & 0 & 0 \\ 0 & \sqrt{D} & 0 & 0 \\ 0 & 0 & \sqrt{D} & 0 \\ 0 & 0 & 0 & \sqrt{D} \end{pmatrix}. \tag{6}$$

From (1) and (5), we see that [27]

$$C(\rho) = g_{00}, \qquad \sqrt{D(\rho)}\rho = ar, \qquad \frac{d\rho}{dr} = \sqrt{\frac{-g_{11}}{D(\rho)}}. \tag{7}$$

Using the general coordinate transformation

$$h^a_\mu = \frac{\partial X'^\nu}{\partial X^\mu} h^a_\nu, \tag{8}$$

where $\{X^\mu\}$ and $\{X'^\nu\}$ are the isotropic and Schwarzschild coordinates, we can get the tetrad in the Schwarzschild coordinates system:

$$h^a_\mu = \begin{pmatrix} \gamma_{00} & 0 & 0 & 0 \\ 0 & \gamma_{11}\sin\theta\cos\phi & ar\cos\theta\cos\phi & -ar\sin\theta\sin\phi \\ 0 & \gamma_{11}\sin\theta\sin\phi & ar\cos\theta\sin\phi & ar\sin\theta\cos\phi \\ 0 & \gamma_{11}\cos\theta & -ar\sin\theta & 0 \end{pmatrix}. \tag{9}$$

It is obvious that the components $(h^1_2, h^1_3, h^2_2, h^2_3, h^3_2)$ change a factor a comparing with those in the Schwarzschild space-time [25].

3 The Interference Phase Along the Light-Ray Trajectory

In a radical direction, there is no angular momentum, the interference phase can be written as [4]

$$\Phi(null) = \int_A^B (E_k dt - p_k dr) = \int_A^B \left(E_k \frac{dt}{dr} - p_k\right) dr. \tag{10}$$

The radical null geodesic is

$$\frac{dr}{dt} = B(r) = \left(a^2 - \frac{2M}{r}\right).$$

From the mass-shell relation $m^2 = g^{\mu\nu} p_\mu^{(k)} p_\nu^{(k)}$, we obtain that [4, 6]

$$p_k(r) = \pm \frac{1}{B(r)} \sqrt{E_k^2 - B(r)m^2}. \tag{11}$$

Thus, the phase is

$$\Phi(null) = \pm \int_A^B \frac{1}{B(r)} \left[E_k - \sqrt{E_k^2 - B(r)m^2}\right] dr. \tag{12}$$

Adopting the relativistic condition, $m \ll E_k$, we have

$$E_k \simeq E_0 + O\left(\frac{m^2}{2E_0}\right), \tag{13}$$

$$\sqrt{E_k^2 - B(r)m^2} \simeq E_k - B(r)\frac{m^2}{2E_0}, \tag{14}$$

where E_0 is the energy at infinity for a massless particle. We can get the phase

$$\Phi(null) = \pm \int_A^B \frac{m^2}{2E_0} dr = \frac{m^2}{2E_0}(r_B - r_A). \tag{15}$$

It is noted that E_0 is the conserved energy of a particle traveling along the geodesic, which is, $(1 - 8\pi\eta^2)E_{0Sch}$, where E_{0Sch} is the energy in the Schwarzschild space-time. Therefore, the neutrino phase scales up by factor $\frac{1}{1-8\pi\eta^2}$ comparing with the value in the Schwarzschild space-time.

4 The Interference Phase Along the Geodesic

In this section, we will calculate the neutrino phase along the radical geodesic. The mass shell condition can be obtained from the geodesic equation:

$$ds^2 = g_{00}dt^2 + g_{11}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{16}$$

The classical orbit can be defined in a plane, $\theta = \frac{\pi}{2}$, $d\theta = 0$, and in a radical direction, $d\varphi = 0$. So, the geodesic equation can be rewritten as

$$g_{00}\dot{t}^2 + g_{11}\dot{r}^2 = 1, \tag{17}$$

where the dot represents the derivative to ds. Substituting (4) into above equation, we have

$$\frac{ds}{dr}=\sqrt{\frac{-g_{00}g_{11}}{(\frac{E_0}{m})^2-g_{00}}}. \tag{18}$$

The phase can be calculated as follows [6]:

$$\begin{aligned}\Phi(geod)&=\pm\int_A^B m\left(\frac{ds}{dr}\right)=\pm\int_A^B\frac{mdr}{\sqrt{(\frac{E_0}{m})^2-g_{00}}}\\&=\pm\left\{\left(\frac{mr_B}{k}\right)\sqrt{k+\frac{2M}{r_B}}-\left(\frac{mr_A}{k}\right)\sqrt{k+\frac{2M}{r_A}}\right.\\&\quad-\left(\frac{m2M}{2k^{3/2}}\right)\log\left[2M+2kr_B+2r_B\sqrt{k\left(k+\frac{2M}{r_B}\right)}\right]\\&\quad\left.+\left(\frac{m2M}{2k^{3/2}}\right)\log\left[2M+2kr_A+2r_A\sqrt{k\left(k+\frac{2M}{r_A}\right)}\right]\right\}\\&\approx\pm\frac{m^2}{E_0}(r_B-r_A)=2\Phi(null),\end{aligned} \tag{19}$$

where

$$k=\left(\frac{E_0}{m}\right)^2-(1-8\pi\eta^2).$$

It is noted that the phase of the geodesic is also scaled up by a factor of $\frac{1}{1-8\pi\eta^2}$, which is the same as the phase of the null.

5 Conclusion and Discussion

In this paper, we have given the interference phase of mass neutrino propagating in the radical direction along the geodesic line and the null line in the space-time with a global monopole which has a topological defect. Monopole formed as a result of a gauge-symmetry breaking is similar to elementary particles. Most of their energy is concentrated in a small region near the monopole core [28]. The global monopole can effect the particle orbits and Hawking radiation (scaling up or scaling down by a factor concerned with gauge-symmetry breaking η). We mainly study the influence of the global monopole on the interference phase of the neutrino. Considering the change of the energy of a particle traveling along the geodesic, we conclude that the neutrino phase in the space-time with a global monopole scales up by a factor, $\frac{1}{1-8\pi\eta^2}$, both for the null and geodesic. The relation $\Phi(geod)=2\Phi(null)$ is still true in this paper, which is in agreement with the result in the Schwarzschild space-time. For a typical grand unification scale, $8\pi\eta^2\ll 1$, the influence of the gauge-symmetry breaking η on the interference phase is small. It is founded that the interference phase scales up more and more as η increases greater and greater. To the extreme condition $\eta\to 1/\sqrt{8\pi}$, the phase tends to become divergent.

The properties of neutrino are required to study deeply in particle physics and cosmology. The establishment of the results of neutrino oscillation will produce great influence on the research of the origin, the evolvement and the final destiny of the universe. Thus, the study on mass neutrino oscillation is more and more important.

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